Varieties of nearrings satisfying $x^n = x$

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Jacobson proved that every ring satisfying $x^n = x$ for some $n > 1$ is a subdirect product of finite fields. Let V_n denote the variety of zero-symmetric nearrings with one-sided identity that satisfy $x^n = x$ $(n > 1)$. Along the lines of Jacobson's result, it can be shown that all subdirectly irreducible elements in V_n are nearfields. Consequently $x + y = y + x$ holds in every V_n. If n is such that every nearfield with $x^n = x$ is a field, then we also have $xy = yx$ in V_n. This leads to the question whether such n exist. For the answer, we have to study the structure of nearfields whose multiplicative groups have finite exponents.

We give results on the finiteness and the numbers of nearfields whose multiplicative groups have exponents $2^k, 3, 6$, or 12. For the proofs, we consider sharply 2-transitive permutation groups.